Available online at www.sciencedirect.com
ScienceDirect
Physica E 40 (2008) 1267–1269
PHYSICA E
www.elsevier.com/locate/physe

Suppression of the Fano factor in nanoelectromechanical systems

Federica Haupt[a,*], Fabio Cavaliere[a], Rosario Fazio[b,c], Maura Sassetti[a]

[a] *Dipartimento di Fisica, Università di Genova and LAMIA INFM-CNR, Via Dodecaneso 33, 16146 Genova, Italy*
[b] *International School for Advanced Studies (SISSA), Via Beirut 2-4, I-34014 Trieste, Italy*
[c] *NEST INFM-CNR and Scuola Normale Superiore, I-56126 Pisa, Italy*

Available online 14 September 2007

Abstract

We analyze the shot noise of a spin-degenerate electronic level coupled to an harmonic oscillator in the presence of relaxation. We show that the electromechanical coupling can induce a suppression of the Fano factor below the value expected without phonons $F = \frac{5}{9}$. Moreover, in the presence of finite relaxation, the Fano factor can be even reduced below $\frac{1}{2}$.

PACS: 73.50.Td; 73.23.−b; 85.85.+j

Keywords: Shot noise; Relaxation; NEMS; Electromechanical coupling

1. Introduction

Nanoelectromechanical systems (NEMS) form a new class of devices which couple electrons motion to mechanical degrees of freedom [1]. Currently, many species of NEMS have been realized and investigated, including single oscillating molecules, semiconductor beams and suspended carbon-nanotubes. In these devices, the tunneling of single electrons creates vibrational excitations and, at finite bias, it may drive the phonon distribution out of equilibrium. In turn, vibrational excitations affect the transport properties of the NEMS and several peculiar transport regimes, such as shuttling instability [2] and avalanche-like transport [3] have been predicted.

In this work we discuss how the coupling between mechanical and electrical degrees of freedom can induce a suppression of the Fano factor in a NEMS. The system we consider is a spin-degenerate single-level coupled to an harmonic oscillator in the sequential tunneling regime. We show that because of the electromechanical coupling, the Fano factor of such a system can be reduced below the value expected for the single-level alone. Moreover, we observe that relaxation of the vibrational excitations enhances the suppression of the Fano factor, and it can even drive the Fano factor below $\frac{1}{2}$, i.e. the minimal value usually expected for a Coulomb blockaded system [4].

2. Model

The system is a spin-degenerate electronic level coupled to an harmonic oscillator and connected to external leads by tunneling barriers. This is described by the Hamiltonian $H = H_s + H_{leads} + H_t$, where ($\hbar = 1$)

$$H_s = \varepsilon n_d + U n_d(n_d - 1)/2 + \sqrt{\lambda}\omega_0(b^\dagger + b)n_d + \omega_0 b^\dagger b,$$

is the Hamiltonian of the NEMS. Here, $n_d = \sum_{\sigma=\uparrow,\downarrow} d_\sigma^\dagger d_\sigma$ is the occupation number of the single level, and U and ε are charging and the single particle energy, respectively. We assume that ε can be tuned by means of an external gate voltage V_g. The parameter $\sqrt{\lambda}$ defines the strength of the electromechanical interaction and $b^\dagger$ creates vibrational excitations of energy ω_0. The leads are Fermi liquids $H_{leads} = \sum_{\alpha=L,R}\sum_{k,\sigma}\varepsilon_{\alpha,k}c_{\alpha,k\sigma}^\dagger c_{\alpha,k\sigma}$ at equilibrium with their chemical potential $\mu_{L,R} = \mu_0 \pm eV/2$, where V is the applied bias voltage. The tunneling coupling is given by $H_t = \sum_\alpha \sum_{k,\sigma}(t_0 c_{\alpha,k\sigma}^\dagger d_\sigma + \text{h.c.})$ where, for simplicity, we assumed symmetric tunneling barriers t_0.

*Corresponding author.
E-mail addresses: haupt@fisica.unige.it, federica.haupt@gmail.com (F. Haupt).

1386-9477/$ - see front matter
doi:10.1016/j.physe.2007.08.090

We consider the weak tunneling limit and treat H_t as a perturbation. It is convenient to diagonalize the unperturbed Hamiltonian by eliminating the coupling term $\sqrt{\lambda}(b^\dagger + b)n_d$. This can be done by means of a canonical polaron transformation [5,6], at the cost of renormalizing the parameters ε and U and the tunneling matrix element $t_0 \to t_0 \exp[\sqrt{\lambda}(b - b^\dagger)]$. From now on, we refer to the renormalized parameters as ε and U.

We consider the regime of strong Coulomb blockade $U \gg \omega_0, eV, k_B T$, where there are only two charge states $n = 0, 1$ involved into transport. If the level broadening induced by tunneling γ is small ($\gamma \ll \omega_0, k_B T$), the dynamics of the system is well described in terms of the classical rate equations:

$$\dot{P}_{nq} = \sum_{\bar{n},\bar{q}} [P_{\bar{n}\bar{q}} \Gamma_{\bar{n},n}^{\bar{q},q} - P_{nq} \Gamma_{n,\bar{n}}^{q,\bar{q}}] + \sum_{\bar{q}=q\pm1} [P_{n\bar{q}} \Gamma_{\text{rel}}^{\bar{q},q} - P_{nq} \Gamma_{\text{rel}}^{q,\bar{q}}],$$

where P_{nq} is the occupation probability of the state with n electrons and q excited phonons in the polaron picture, and $\Gamma_{n,\bar{n}}^{q,\bar{q}} = \sum_\alpha \Gamma_{\alpha n,\bar{n}}^{\ q,\bar{q}}$ are the golden-rule tunneling rates

$$\Gamma_{\alpha 0,1}^{\ q,\bar{q}} = 2\Gamma_0 X_{q\bar{q}} f(\varepsilon + \omega_0(\bar{q} - q) - \mu_\alpha),$$
$$\Gamma_{\alpha 1,0}^{\ q,\bar{q}} = \Gamma_0 X_{q\bar{q}} [1 - f(\varepsilon + \omega_0(q - \bar{q}) - \mu_\alpha)]. \tag{1}$$

Here, $\Gamma_0 = 2\pi\nu t_0^2$ is bare tunneling rate (ν density of states of the leads), $f(x)$ is the Fermi function, and $X_{q\bar{q}} = |\langle q| \exp[\sqrt{\lambda}(b - b^\dagger)]|\bar{q}\rangle|^2$ are the Franck–Condon factors [3] responsible for the highly non-trivial dependence on $q, \bar{q}$ of the tunneling rates. The factor 2 in the rates for entering into the level stems from spin degeneracy [9].

In rate equations we have also introduced the relaxation rates

$$\Gamma_{\text{rel}}^{q,q-1} = e^{\beta\omega_0} \Gamma_{\text{rel}}^{q-1,q} = q\Gamma_0^{\text{rel}}/(1 - e^{-\beta\omega_0}) \tag{2}$$

which represent transitions between neighboring phonon states induced by the coupling of the harmonic oscillator to an external dissipative bath [7]. In the following, we introduce the adimensional parameter $w = \Gamma_0^{\text{rel}}/\Gamma_0$ to define the strength of relaxation.

Within the rate equation approach, the steady current $\langle I \rangle$, the zero-frequency current noise S and the Fano factor $F = S/2e\langle I \rangle$ can be evaluated by means of standard techniques [8].

3. Results

In this paper, we consider the behavior of the Fano factor F in the presence of coupling to phonons. It is well known that strong coupling ($\lambda \gg 1$) can lead to a huge enhancement of F [3]. Our aim is instead to investigate regimes in which the electromechanical coupling helps to *reduce* the Fano factor. As a reference term, we take the value of F in a phononless system ($\lambda = 0$). At low temperatures it is simply given by [9]:

$$F_{\lambda=0} = (\Gamma_{\text{in}}^2 + \Gamma_{\text{out}}^2)/(\Gamma_{\text{in}} + \Gamma_{\text{out}})^2, \tag{3}$$

where $\Gamma_{\text{in (out)}}$ are the rates for tunneling in and out of the level and correspond to the limits of $\Gamma_{n,\bar{n}}^{q,\bar{q}}$ for $\lambda = 0$ (note $X_{q\bar{q}} = \delta_{q\bar{q}}$ for $\lambda = 0$). Clearly, it is $\frac{1}{2} \leqslant F_{\lambda=0} \leqslant 1$, and in particular $F_{\lambda=0} \equiv \frac{5}{9}$ in the transport regime, being $\Gamma_{\text{in}} = 2\Gamma_{\text{out}}$.

In the presence of coupling to phonons, $\lambda \neq 0$, the dynamics of the system becomes highly complex and, in general, there is no analytic expression for the Fano factor, so that one has to resort to extensive numerical investigation.

Let us first consider the case of no relaxation $w = 0$. We notice that while for $\varepsilon \leqslant 0$ it is always $F \geqslant \frac{5}{9}$, for $\varepsilon > 0$ it is possible to have a Fano factor below $\frac{5}{9}$ (e.g. Fig. 1a). This suppression can be understood observing that the limit $F = \frac{5}{9}$ is essentially a consequence of spin degeneracy, which induces an intrinsic asymmetry between the rates for tunneling in and out of the level, see Eqs. (1). However, for $\varepsilon > 0$ there are more energetically allowed transitions for leaving the level than for entering. This fact, combined with the peculiar dependence of the factors $X_{q\bar{q}}$ on the vibrational indices $q, \bar{q}$, can partially compensate the asymmetry due to the spin, giving $F < \frac{5}{9}$.

In the presence of relaxation ($w \neq 0$), the situation becomes even more interesting. Infact, we have found that F exhibits a *non-monotonous* dependence on the strength of relaxation w, and that for *any* given λ, there are regions of the (ε, eV)-plane where F is *suppressed below* $\frac{1}{2}$ in certain range of w (e.g. Fig. 1b, c). Moreover, there exists a

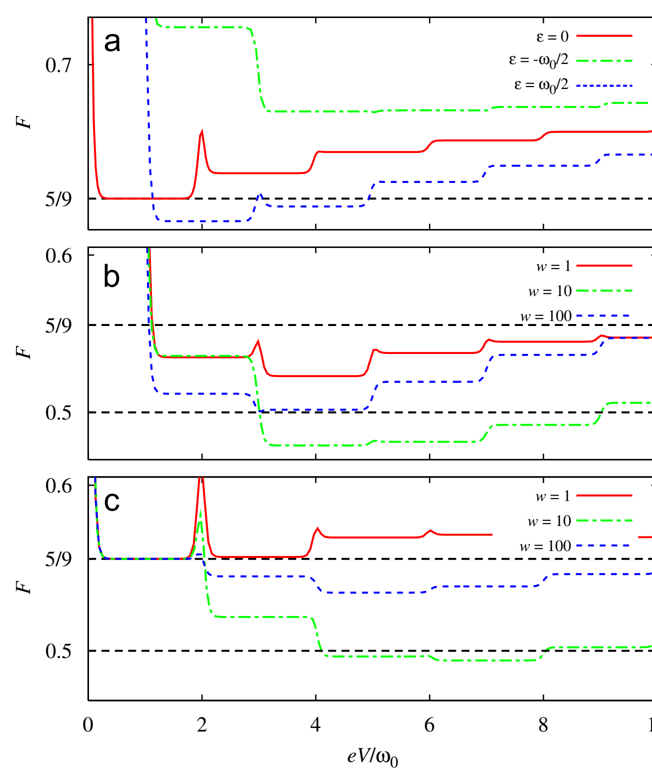

Fig. 1. Fano factor F as a function of voltage V (units ω_0/e) for $\lambda = 1.4$. (a) F for $w = 0$ and different the energies of the level: $\varepsilon = 0$ (level on-resonance) and $\varepsilon = \pm\omega_0/2$. (b) F at fixed $\varepsilon = \omega_0/2$ for different values of the relaxation strength w. (c) Same as in b but for $\varepsilon = 0$. In all panels, $E_c = 10\,\omega_0$ and $k_B T = 0.02\,\omega_0$.

λ-dependent upper bound w_λ such that for $w > w_\lambda$, the Fano factor is always larger than $\frac{1}{2}$ and tends rapidly to the value it would have if the phonons were at thermal equilibrium [7]. This is because for $w > w_\lambda$, the electronic and the mechanical degrees of freedom tend to decouple and the dynamics of the system reduces to an effective two-state process [6]. The existence of w_λ then strongly support the idea that the suppression of the Fano factor below $\frac{1}{2}$ is the result of the dynamical competition between tunneling and relaxation.

It is worth to remember that the Fano factor of a Coulomb blockaded system in the sequential tunneling regime is generally larger or equal to $\frac{1}{2}$ [4]. As remarkable exceptions to this "rule of thumb" we mention the strong suppression of F predicted in the shuttling regime [2], and the recent measurements of Fano factors below $\frac{1}{2}$ in a quantum dot induced by the finite bandwith of the detector [10].

In our case, the suppression of the Fano factor is rather a dynamical effect given by the interplay between vibration-assisted tunneling and phonon relaxation, which induces correlations between different current pulses via emission–absorption of phonons.

Finally, let us shortly consider the case $\varepsilon = 0$ (level on-resonance). In this case, the electromechanical coupling alone is not sufficient to give $F < \frac{5}{9}$. However, for any value of λ, F can be suppressed below $\frac{5}{9}$ as a consequence of relaxation (e.g. Fig. 1c). The situation is different when we consider the suppression of the Fano factor below $\frac{1}{2}$. Infact we have found that it is possible to achieve $F < \frac{1}{2}$ at $\varepsilon = 0$ only for $\lambda > 1$.

In conclusion, we considered the Fano factor of a spin-degenerate electronic level coupled to an oscillator in the presence of an dissipative bath. We have showed that because of the electromechanical coupling, the Fano factor can be suppressed below the value expected without phonons $F = \frac{5}{9}$. Moreover, in the presence of relaxation the Fano factor can be suppressed even below $\frac{1}{2}$ as a consequence of a dynamical interplay between tunneling and emission–absorption of phonons.

Acknowledgments

This work was supported by the EU via Contact No. MCRTN-CT2003-504574 and by the Italian MIUR via PRIN05.

References

[1] M. Blencowe, Phys. Rep. 395 (2004) 159.
[2] T. Novotný, A. Donarini, C. Flindt, A.-P. Jauho, Phys. Rev. Lett. 92 (2004) 248302.
[3] J. Koch, F. von Oppen, A.V. Andreev, Phys. Rev. B 74 (2006) 205438.
[4] Y.M. Blanter, M. Büttiker, Phys. Rep. 336 (2000) 2.
[5] A. Mitra, I. Aleiner, A.J. Millis, Phys. Rev. B 69 (2004) 245302.
[6] S. Braig, K. Flensberg, Phys. Rev. B 68 (2003) 205324.
[7] F. Haupt, F. Cavaliere, R. Fazio, M. Sassetti, Phys. Rev. B 74 (2006) 205328.
[8] A.N. Korotkov, Phys. Rev. B 49 (1994) 10381.
[9] Yu.V. Nazarov, J.J.R. Struben, Phys. Rev. B 53 (1996) 15466.
[10] S. Gustavsson, R. Leturcq, T. Ihn, K. Ensslin, M. Reinwald, W. Wegscheider, Phys. Rev. B 75 (2007) 075314.